

# Advanced Statistical Methods for Forest Products Manufacturers

A Customized and Practical Training Course  
for  
Forest Products Manufacturing

by

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The University of Tennessee Forest Products Center is located on the Agricultural campus in Knoxville, Tennessee. The Center is housed in the Department of Forestry, Wildlife and Fisheries and functions under the research mission of the Institute of Agriculture Experiment Station.

The mission of the Forest Products Center is to solve problems for forest products producers and provide leadership in research and education to ensure future competitiveness and sustainability of the industry. The Center is focused on providing research and education for the forest products industry in Tennessee, the region, and beyond.

The Tennessee Agricultural Experiment Station conducts mission oriented research programs to serve Tennessee agriculture and forestry producers. The Center is an outgrowth of the Governor's Council on Agriculture and Forestry, convened in May 1996. The Center is the only such full-time research facility among U.S. university programs. Undergraduate and graduate students are included in the research programming of the Center and gain valuable experience in conducting research.



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