Bootstrap confidence intervals for percentiles of reliability data for wood-plastic composites

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Abstract

Improving product reliability is an important goal that may be achieved from a better understanding of the product's lower percentiles. These lower percentiles provide practitioners with an evaluation of the product's early failures along with providing information for specification limits, warranty, and cost analysis. Estimation of lower percentiles is sometimes difficult, since substantive data are often sparse in the lower tails. Bootstrap techniques provide important solutions for confidence interval evaluations of these percentiles. This paper applies three bootstrapping methods to appraise the modulus of elasticity (MOE) and modulus of rupture (MOR) test results on sampled wood plastic composites (WPC). The fully nonparametric, the fully parametric, and the nonparametric bootstrap for parametric inference (NBSP) bootstrapping methods for the MOE and MOR of WPC were assessed. For each of the three methods, three different types of confidence intervals, including the standard, percentile, and bias-corrected intervals were evaluated. For smaller sample sizes, the fully nonparametric bootstrapping method was less desirable than the fully parametric or NBSP methods for smaller percentiles. The fully parametric method cannot be used when the censoring technique is unknown. These bootstrapping methods may benefit WPC manufacturers with a better understanding of material properties by providing warning signs of poor reliability.

Meeker and Escobar (1998) remarked that traditional parameters of a statistical model (e.g., mean and SD) are not of primary interest. Instead, design engineers, reliability engineers, managers, and customers are interested in specific measures of product reliability or particular characteristics of a failure-time (or failure-pressure) distribution (e.g., failure probabilities, quantiles of the life distribution, failure rates, etc.).

This paper focuses on estimating percentiles of material properties with an emphasis on the lower percentiles that relate to product failure. These estimation procedures may also be applied more generally with many other parameters of interest in wood science. The objective of the research was to estimate the lower percentiles of the material properties using nonparametric bootstrapping methods. Wood-plastic composites data are used as an example even though bootstrapping methods can be applied to any data set. The bootstrapping methods outlined in this paper can be applied to generating tolerance limits for product design and may also be helpful to the practitioner in ensuring product quality in the lower percentiles of material properties.

Reliability measurements using any parameters (e.g., percentiles), must acknowledge statistical variation so that product improvements may be realized. Thus, wood scientists, supervisors, and line workers need realistic and robust

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confidence intervals to assess potential variability in material properties. Such confidence intervals may also minimize the risk of error in decision making. We do not want to assume normality of data, since in many cases this and other assumptions will not hold in many wood science settings.

Edwards at al. (2008) note that historically, the problem of estimating percentiles was not in finding point estimators, but in finding standard errors and thus, confidence intervals of percentiles. Serfling (1980) thoroughly examines the asymptotic distribution of the sample quantiles. In particular, under mild requirements (i.e., smoothness of the distribution function), the sample quantiles are asymptotically normal. The normal approximate confidence interval for $\hat{t}_p$ is given by:

$$\hat{t}_p \pm z^{1-\alpha/2}\widehat{se}_{\hat{t}_p} \quad [1]$$

where $\widehat{se}_{\hat{t}_p}$ is the standard error of the estimate approximated by

$$\widehat{se}_{\hat{t}_p} = \sqrt{\widehat{Var}(\hat{t}_p)} = \hat{t}_p\{\widehat{Var}(\hat{\mu}) + 2\Phi^{-1}(p)\widehat{Cov}(\hat{\mu},\hat{\sigma}) + [\Phi^{-1}(p)]^2\widehat{Var}(\hat{\sigma})\}^{1/2} \quad [2]$$

which is derived using the delta method and Φ^{-1} represents the inverse of the cumulative standardized location-scale distribution. Standardized location-scale distributions are actually a general class of distributions that include the normal, t-, and stable distributions, as well as monotonic transformations of these distributions, such as the log (Bodurtha and Shen 2004). $\widehat{Var}(\hat{\mu})$, $\widehat{Var}(\hat{\sigma})$, and $\widehat{Cov}(\hat{\mu},\hat{\sigma})$ are obtained from the variance-covariance matrix or inverse Fisher information matrix, $\mathcal{F}^{-1}$.

When the sample size is sufficiently large, these asymptotic normal intervals can provide sound approximations. In many industrial and laboratory settings, large enough samples for parametric-based approximations are too costly or too time consuming to obtain. Bootstrap methods, however, are more realistic and may yield valid results not requiring parametric models. Bootstrap methods may also be used with various parametric models. Bootstrap techniques may provide empirical validation on whether the normal approximation is appropriate for both the parametric and nonparametric models.

As Edwards et al. (2008) further note, bootstrapping is a computer intensive statistical method where the basic idea is to simulate the sampling process a specified (usually large) number of times and obtain an empirical bootstrap distribution for a desired population parameter. This empirical bootstrap distribution is then used to acquire characteristics (e.g., standard error, bias estimates, confidence intervals, etc.) about the estimated parameter. This paper describes bootstrap sampling methods and confidence intervals of percentiles for wood plastic composites. Bootstrap sampling methods and confidence intervals of percentiles are then examined in greater detail for both modulus of elasticity (MOE) and modulus of rupture (MOR) data for WPC. Although bootstrap methods are described for WPC, these same methods may be utilized more generally for many types of engineering, quality, and reliability estimations.

We believe the bootstrap method adds value to wood scientists by providing methods that may be more defendable in reporting research results when asymptotic assumptions are not valid. Bootstrap methods may also provide value to practitioners in minimizing the risk of making incorrect business decisions.

Materials and methods

Wood-plastic composites data set

Product test data examined in this paper include modulus of rupture (MOR) and modulus of elasticity (MOE) of WPC. The MOR is defined as the maximum stress at failure. The MOE is defined as the rate of change of strain as a function of stress. Perhac (2007) and Perhac et al. (2007) explored classical reliability methods of the MOE and MOR for WPC. They found that the addition of a copolymer-coupling agent, malienated polypropylene (maleic anhydride modified polypropylene or MAPP), in the production process increases MOR but has no substantial effect on the MOE.

Both the MOE and MOR data sets with MAPP and the MOR data without the outlier were used for this analysis. The outlier was determined as an error in the data that occurred with the testing device during measurement. Units are reported in megapascals (MPa). The time interval between destructive test samples varied between 1 and 4 hours; therefore consideration for autocorrelation was not warranted. The assumption of normality is not valid for either MOR or MOE of this WPC data, see Perhac (2007). Similar work has been performed on medium density fiberboard (MDF) by Young and Guess (2002), Guess et al. (2003), Guess et al. (2004), Chen et al. (2006), and Edwards et al. (2008). However, the aforementioned literature relied on industrial-based data sets for non-WPC and did not conduct a designed experiment on the effect that product additives may have on the lower percentiles of material properties for WPC, with specific estimates of bootstrap confidence intervals for these lower percentiles. Bootstrap intervals for the same sample size can have smaller standard errors, and thus better precision. This would also allow for a smaller sample size with the same precision as compared to parametric confidence intervals that are non-bootstrap based. This adds great value to WPC. The authors are not aware of any published literature as related to this topic for WPC.

The bootstrapping methods used in this paper may be utilized to better understand the lower percentiles of MOR and MOE measures, helping practitioners improve the understanding of reliability of WPC. Bootstrapping can also be quite helpful for many other measurements needed in the wood sciences.

Bootstrap methods and confidence intervals

Helpful reviews of bootstrap methods are Efron and Tibshirani (1994), and DiCiccio and Efron (1996). Meeker and Escobar (1998) noted that the "justification for the bootstrap is based on large-sample theory." But, even when large samples are used, there can still be problems with the tails of the sample, e.g., lower percentiles. The sampling distribution obtained from the bootstrap may not be continuous, resulting in the calculation of inaccurate confidence intervals. Chernick (1999) discusses additional limitations.

Three different bootstrap methods are used in this analysis (Edwards et al. 2008). The first and most basic bootstrap is the fully nonparametric bootstrap (Martinez and Martinez 2002). The steps of this procedure are described as follows. For a random sample or a given data set, $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ of size n, any population parameter ϑ is estimated by $\hat{\theta}$, e.g., mean, variance, etc. Sampling is then done with replacement from the original data set to obtain a bootstrap sample of size n, denoted by $\mathbf{x^{*b}} = (x_1^{*b}, x_2^{*b}, \ldots x_n^{*b})$. Resampling with replacement is done a large number of times, B. The same statistic as above is calculated from each bootstrap sample and is denoted as $\hat{\theta}^{*b}$, where b represents the b^{th} bootstrap sample. The empirical bootstrap distribution of $\hat{\theta}^*$ is then defined and used as an estimate for the distribution of $\hat{\theta}$. The advantage of this sampling method is that the underlying distribution does not have to be known or assumed.

The second bootstrapping method used in this paper is the fully parametric bootstrap, which requires knowledge of a parametric or underlying distribution. This method is similar to the nonparametric method described above except that with the fully parametric method, bootstrap samples are taken from the given parametric distribution using the same maximum likelihood estimates taken from the original sample. Efron and Tibshirani (1994), Meeker and Escobar (1998), and Chernick (1999) describe the fully parametric bootstrap. Meeker and Escobar (1998) stated an important disadvantage of the fully parametric method. Since data are simulated, the censoring process must be completely specified for reliability data. Meeker and Escobar (1998), however, caution this will be "more difficult for complicated systematic or random censoring." This specification is simple for complete data, such as the MOE and MOR data used in this analysis.

The third and final bootstrapping method used in this analysis is the nonparametric bootstrap sampling method for parametric inference (Meeker and Escobar 1998), which is conveniently denoted as NBSP (Edwards 2004, Edwards et al. 2008). Similar to the fully nonparametric method, the NBSP method samples with replacement from the original data, but for each bootstrap sample of size n, maximum likelihood estimates are realized from the specified parametric model. These maximum likelihood estimates are then used to estimate the population parameter of interest and form the bootstrap distribution.

Different methods are available for the construction of bootstrap confidence intervals for population parameters. This paper utilizes three methods: the standard normal bootstrap confidence interval, the bootstrap percentile interval, and the bias-corrected bootstrap percentile interval. Theoretical details of these methods are omitted. For a complete discussion, compare Efron and Tibshirani (1994), DiCiccio and Efron (1996), and Davison and Hinkley (1997). Edwards (2004) and Edwards et al. (2008) give detailed algorithms for each of the bootstrap confidence intervals used. The actual intervals are defined as follows.

The bootstrap standard confidence interval is given by:

$$[\hat{\theta} - z^{(\alpha/2)}\widehat{se}_{\hat{\theta}},\ \hat{\theta} + z^{(1-\alpha/2)}\widehat{se}_{\hat{\theta}}] \qquad [3]$$

Note that $\widehat{se}_{\hat{\theta}}$ is the standard error found by calculating the SD of the bootstrap estimates of θ, and $z^{(\alpha/2)}$ is the $\alpha/2th$ quantile of the standard normal distribution.

The bootstrap percentile confidence interval is based upon the quantiles of the bootstrap distribution of estimates and is given by:

$$[\hat{\theta}^{*(\alpha/2)},\ \hat{\theta}^{*(1-\alpha/2)}] \qquad [4]$$

where $\hat{\theta}^{*(\alpha/2)}$ and $\hat{\theta}^{*(1-\alpha/2)}$ are actually the quantiles of the bootstrap distribution of estimates.

Finally, the bias-corrected percentile interval is defined as the amount of difference between the median of the bootstrap estimates $\hat{\theta}^{*b}$ and the estimate, $\hat{\theta}$ from the original sample

Table 1. — 95 percent asymptotic normal confidence intervals for WPC modulus of elasticity (MOE) with coupling agent (MAPP).

p	$\hat{t}_p$ = quantile	LCL	UCL
0.01	2476.5	2265.5	2687.5
0.05	2894.0	2725.9	3062.1
0.10	3116.5	2968.6	3264.4
0.50	3901.5	3791.9	4011.1

(Efron 1981, 1987). The bias correction constant estimate, denoted by $\hat{z}_0$, is defined as:

$$\hat{z}_0 = \Phi^{-1}\left(\frac{\#(\hat{\theta}^{*b} < \hat{\theta})}{B}\right) \qquad [5]$$

where Φ^{-1} symbolizes the inverse cumulative normal distribution and # means "number of."

Then, a $100(1-\alpha)$percent bias-corrected percentile confidence interval for θ is given by:

$$[\hat{\theta}^{*(\alpha_1)},\ \hat{\theta}^{*(\alpha_2)}] \qquad [6]$$

where α_1 and α_2 are the revised quantities on which to base the percentile confidence interval endpoints. These quantities are defined as:

$$\alpha_1 = \Phi(2\hat{z}_0 + z^{(\alpha/2)}) \qquad [7]$$

and

$$\alpha_2 = \Phi(2\hat{z}_0 + z^{(1-\alpha/2)}) \qquad [8]$$

where Φ is the cumulative standard normal distribution.

Helpful comments when samples are rather small are found in several sources (Meeker and Escobar 1998, Chernick 1999, Polansky 2000).

Results and discussion

Percentile bootstrap confidence intervals for the MOE of WPC

For each bootstrap method of sampling, the standard normal, percentile, and bias-corrected percentile bootstrap intervals were constructed and compared for the 1st, 5th, 10th, and 50th percentiles for the MOE and MOR for WPC. Sample sizes are small for each of the original data sets, with the sample size for MOE equal to 120, and the sample size for MOR equal to 119. Meeker and Escobar (1998) recommended that the number of bootstrap samples be between 2000 and 4000 in order to construct confidence intervals, especially for the lower percentiles. To expand upon the work of Meeker and Escobar (1998), for each sampling method for the MOE and MOR data sets, 5000 bootstrap samples of the same size as the original sample were created. More discussion of the number of bootstrap samples is given in the Results and Discussion section. The asymptotic normal confidence intervals will also be provided in order to compare with the bootstrap results. The MATLAB code used can be downloaded at www.spcforwood.com.

Table 1 provides the 95 percent asymptotic normal confidence intervals for the MOE of WPC. The fully nonparametric MOE 95 percent bootstrap confidence intervals are shown in **Table 2** while the fully parametric 95 percent bootstrap confidence intervals are displayed in **Table 3**. Finally, the

Table 2. — *Fully nonparametric 95 percent bootstrap confidence intervals for WPC modulus of elasticity (MOE) with coupling agent (MAPP).*

p	$\hat{t}_p$ = quantile	Interval type	LCL	UCL
0.01	2675.4203	Standard	2537.8457	2803.1368
		Percentile	2583.7283	2812.4689
		Bias-Corrected	2583.7283	2723.6216
0.05	2879.3770	Standard	2700.8193	3048.4685
		Percentile	2730.4557	3025.3233
		Bias-corrected	2729.9046	3012.4229
0.10	3026.3612	Standard	2925.3570	3120.1178
		Percentile	2918.0135	3121.3363
		Bias-corrected	2886.7757	3090.0984
0.50	4009.9921	Standard	3853.4747	4166.5488
		Percentile	3848.3208	4177.4177
		Bias-corrected	3845.3682	4170.9650

Table 3. — *Fully parametric 95 percent bootstrap confidence intervals for WPC modulus of elasticity (MOE) with coupling agent (MAPP).*

p	$\hat{t}_p$ = quantile	Interval type	LCL	UCL
0.01	2266.4550	Standard	2039.0638	2472.5851
		Percentile	2050.6568	2484.7207
		Bias-corrected	2029.9188	2466.5567
0.05	2806.4363	Standard	2610.5010	2993.7206
		Percentile	2613.6001	2994.8302
		Bias-corrected	2597.7361	2982.9236
0.10	3090.5001	Standard	2915.0694	3252.4072
		Percentile	2917.0026	3254.0456
		Bias-corrected	2902.5000	3241.5136
0.50	3963.4367	Standard	3851.8431	4072.4052
		Percentile	3852.3194	4072.2753
		Bias-corrected	3847.8997	4069.0436

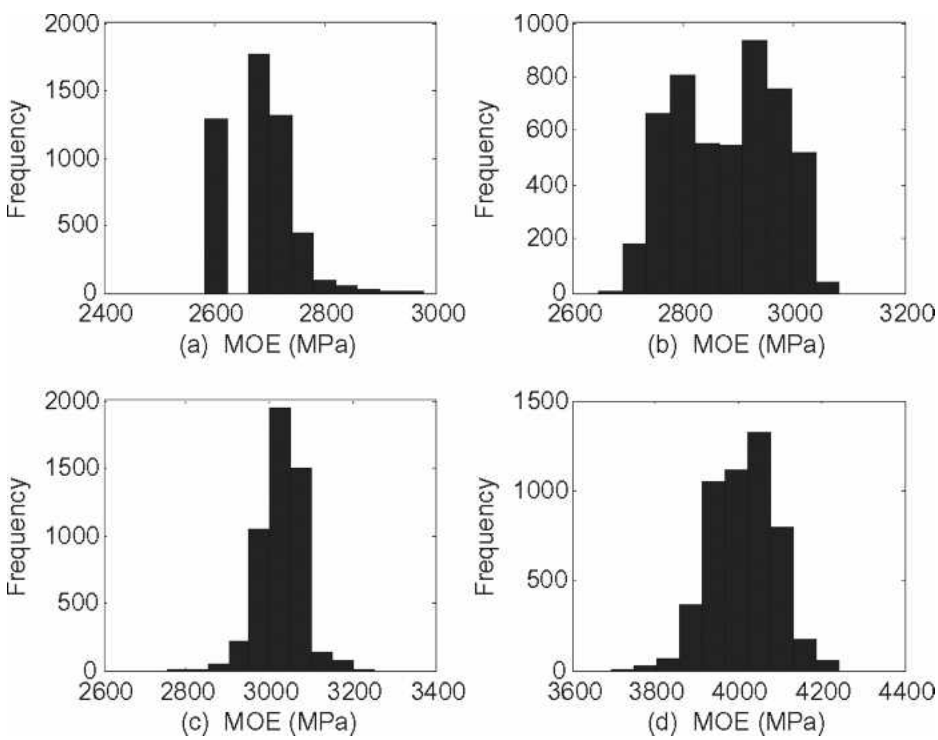

Figure 1. — Sampling distribution of percentiles for WPC MOE with coupling agent (MAPP) under the fully nonparametric bootstrap sampling method. (a) 1st, (b) 5th, (c) 10th, and (d) 50th.

NBSP 95 percent bootstrap confidence intervals are displayed in **Table 4**. In the following tables, LCL represents the lower confidence limit and UCL represents the upper confidence limit. Empirical bootstrap sampling distributions for each of the four quantiles corresponding with each bootstrap interval listed above are shown in **Figures 1**, **2**, and **3**, respectively.

The bootstrap sampling distribution shown in **Figure 1** indicates a constraint of the fully nonparametric bootstrap method. Because the sample size is relatively small for the MOE, the sampling distribution for the first percentile shown in **Figure 1**(a) appears discrete or "snaggle-toothed." This discreteness would be even more apparent if the bootstrap resampling size had been smaller, i.e., less than 5000. When this pattern is observed, practitioners should attempt other bootstrapping methods for the lower percentiles, if possible, such as the NBSP or fully parametric methods. In addition, the first percentile fully nonparametric sampling distribution is shown as heavily right skewed. Increasingly symmetrical and continuous distributions emerge with increasing percentiles, as shown in **Figures 1(b)**, **(c)**, and **(d)**. As shown in **Table 2**, the standard interval for the first percentile is extremely wide, ranging from 2537.85 to 2803.14 MPa. The percentile interval cuts the distribution of the bootstrapped estimates at the 2.5 and 97.5 percentiles to produce a confidence interval of [2583.73, 2812.47] MPa. This percentile interval, however, does not incorporate the right-skewness of the first-percentile distribution. The bias-corrected interval corrects for any right skewness and gives an even smaller confidence interval of [2583.73, 2723.62] MPa. Note that the percentile interval and the bias-corrected percentile interval have the same lower bound of 2583.73, but different upper bounds as a result of the strong right skewness. Recall this is what can happen in the very similar case of Chi-square lower percentile bounds vs. upper percentiles bounds (e.g., see a Chi-square table with smaller degrees of freedom). Identical values for the lower bounds of bootstrap intervals may also be seen with the upper quantiles when the distributions are skewed. An extensive analysis with the fully nonparametric bootstrap sampling method was performed in order to verify these results. We found that the most common scenario (24 of 36 runs) with B = 5000 bootstrap samples each, indicated identical values between the percentile and the bias-corrected intervals for only the first percentile lower confidence limits. The confidence intervals for these 24 tables were similar in both values and widths. The next most

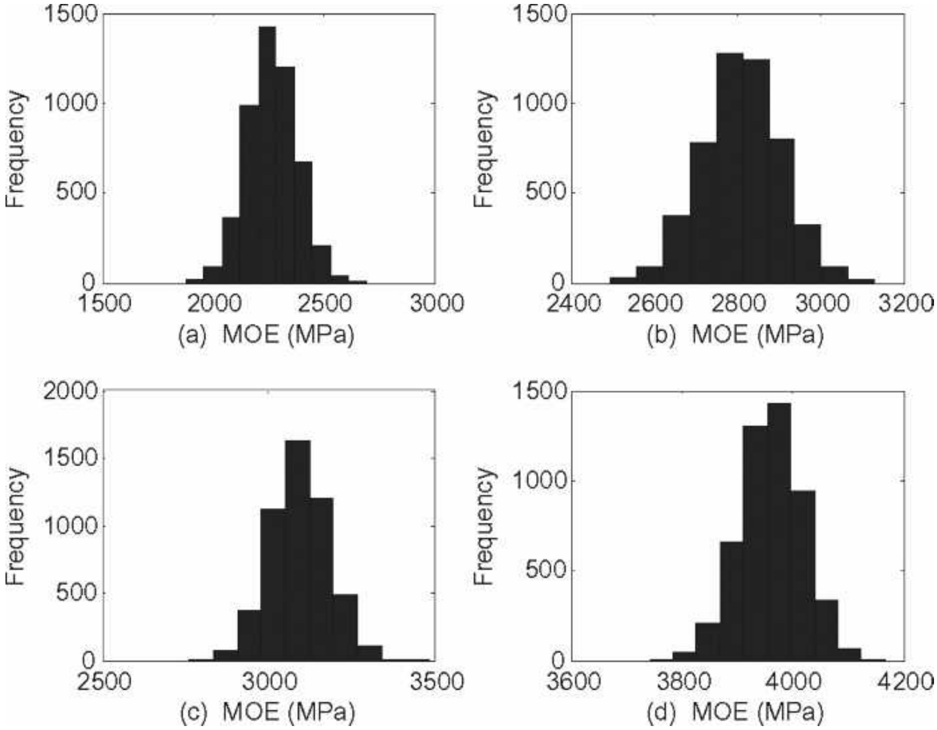

Figure 2. — Sampling distribution of percentiles for WPC MOE with coupling agent (MAPP) under the fully parametric bootstrap sampling method. (a) 1st, (b)5th, (c) 10th, and (d) 50th.

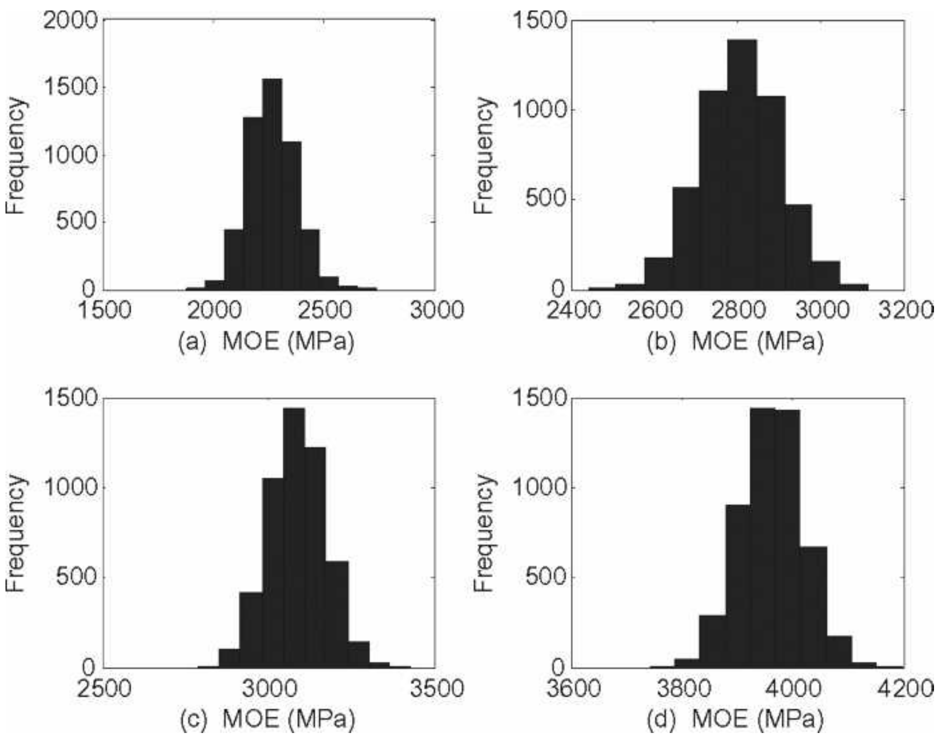

Figure 3. — Sampling distribution of percentiles for WPC MOE with coupling agent (MAPP) under the NBSP method. (a) 1st, (b) 5th, (c) 10th, and (d) 50th.

common scenario, based on six of 36 runs, was that of having identical values between the percentile and the bias-corrected intervals for both the first and fifth percentile lower confidence limits. We found that identical values for the first percentile lower confidence limits are always obtained for the percentile and bias-corrected intervals for the fully nonparametric bootstrap method. We randomly chose one representative table, shown as **Table 2**, from the 24 most commonly presented tables of intervals for the fully nonparametric bootstrap method to include in this paper.

The best underlying parametric distribution for the MOE of WPC was previously determined as Weibull followed by the smallest extreme value and normal distributions (Perhac 2007, Perhac et al. 2007). This best model determination was used to construct the confidence intervals based on both the parametric and the NBSP methods.

Fully parametric 95 percent bootstrap confidence intervals for the MOE are shown in **Table 3**. Unlike the fully nonparametric intervals, widths of the fully parametric intervals are agreeable for each percentile and more representative of the asymptotic normal confidence interval widths. For instance, first-percentile widths of the asymptotic normal (**Table 1**), fully nonparametric standard (**Table 2**), and fully parametric standard (**Table 3**) confidence intervals are 422, 265, and 434, respectively. **Figure 2** shows that the sampling distributions for the fully parametric bootstrap sampling method are much more continuous as well as much more normally distributed than for the fully nonparametric bootstrap method.

The NBSP 95 percent bootstrap confidence intervals are displayed in **Table 4**. These intervals are very similar to those of the fully parametric method with respect to the LCL and UCL limits. In addition, as with the fully parametric method, interval widths for each of the percentiles for the three different types of intervals are very close. Furthermore, LCL and UCL limit values and interval widths are similar to those for the asymptotic normal confidence intervals. **Figure 3** shows the sampling distributions of the percentiles for the NBSP method. Although these sampling distributions are continuous, the normal distribution is not approximated as well as with the fully parametric method. For instance, slight right skewness is shown for the fifth percentile while minor left skewness is evident in the tenth percentile sampling distribution. However, using the NBSP method (Meeker and Escobar

Table 4. — NBSP 95 percent bootstrap confidence intervals for WPC modulus of elasticity (MOE) with coupling agent (MAPP).

p	$\hat{t}_p$ = quantile	Interval type	LCL	UCL
0.01	2265.2672	Standard	2048.5655	2463.0835
		Percentile	2058.6735	2472.5756
		Bias-corrected	2042.4576	2458.1040
0.05	2810.4823	Standard	2617.7072	2986.5144
		Percentile	2629.9603	2997.3603
		Bias-corrected	2618.7599	2982.8038
0.10	3085.9556	Standard	2918.2460	3249.2307
		Percentile	2926.7648	3257.2224
		Bias-corrected	2922.7587	3252.3137
0.50	3961.9834	Standard	3849.7168	4074.5314
		Percentile	3845.2588	4069.7234
		Bias-corrected	3842.7912	4068.7148

Table 5. — 95 percent asymptotic normal confidence intervals for WPC modulus of rupture (MOR) with coupling agent (MAPP).

p	$\hat{t}_p$ = quantile	LCL	UCL
0.01	31.938	30.166	33.710
0.05	35.429	34.017	36.841
0.10	37.290	36.048	38.533
0.50	43.856	42.936	44.777

1998) is an appropriate choice when the sample size is small and there is confidence in the underlying parametric model. This method requires a parametric assumption, but resampling is done with replacement from the original data. The main advantage of using the NBSP method over the fully parametric method is that no censoring assumptions are required; this is important for reliability data when the censoring mechanism is not known.

Percentile bootstrap confidence intervals for MOR of WPC

Table 5 provides the 95 percent asymptotic normal confidence intervals for the MOR of WPC. The fully nonparametric MOR 95 percent bootstrap confidence intervals are shown in **Table 6** while the fully parametric 95 percent bootstrap confidence intervals are displayed in **Table 7**. The NBSP 95 percent bootstrap confidence intervals are displayed in **Table 8**. Empirical bootstrap sampling distributions for each of the four quantiles corresponding with each bootstrap interval listed above are shown in **Figures 4**, **5**, and **6**, respectively.

The nonparametric bootstrap sampling distributions shown in **Figure 4** for the MOR data indicate severe right skewness for the first percentile and severe left skewness for the 10th percentile. Each of the four quantile sampling distributions appears widely dispersed, implying large sampling distribution variability and a lack of normality. As shown in **Table 6**, confidence interval widths become more consistent for three interval types with increasing percentile. Although the interval widths do not necessarily decrease with increasing percentile for the MOR data, the variation of interval width for the three interval types within each percentile does decrease with increasing percentile. For instance, the standard interval for the first percentile ranges from 28.54 to 31.92 MPa. The standard intervals for the fifth, 10th, and 50th percentiles are [30.61, 36.76], [34.75, 39.46], and [43.57, 46.14] MPa, respectively. Average interval differences and the SDs of the three interval types for the 1st, 5th, 10th, and 50th percentiles are 2.470±1.161, 5.882±0.239, 4.641±0.186, and 2.502±0.059, respectively.

Table 6. — Fully nonparametric 95 percent bootstrap confidence intervals for WPC modulus of rupture (MOR) with coupling agent (MAPP).

p	$\hat{t}_p$ = quantile	Interval type	LCL	UCL
0.01	30.7191	Standard	28.5386	31.9247
		Percentile	29.9419	32.8026
		Bias-corrected	29.9419	31.1064
0.05	34.0600	Standard	30.6072	36.7648
		Percentile	31.4409	37.1802
		Bias-corrected	31.1451	36.8933
0.10	37.1516	Standard	34.7542	39.4632
		Percentile	34.4382	38.8692
		Bias-corrected	33.9534	38.7374
0.50	44.6613	Standard	43.5706	46.1406
		Percentile	43.0686	45.5351
		Bias-corrected	43.0005	45.4689

Table 7. — Fully parametric 95 percent bootstrap confidence intervals for WPC modulus of rupture (MOR) with coupling agent (MAPP).

p	$\hat{t}_p$ = quantile	Interval type	LCL	UCL
0.01	29.8483	Standard	27.7424	31.7945
		Percentile	27.8880	31.8891
		Bias-corrected	27.7502	31.7687
0.05	34.8056	Standard	33.0784	36.4104
		Percentile	33.1728	36.4534
		Bias-corrected	33.0446	36.3655
0.10	37.2585	Standard	35.7441	38.6534
		Percentile	35.7424	38.6603
		Bias-corrected	35.5979	38.5295
0.50	44.4793	Standard	43.5809	45.3682
		Percentile	43.5567	45.3583
		Bias-corrected	43.5338	45.3412

Table 8. — NBSP 95 percent bootstrap confidence intervals for modulus of rupture (MOR) with coupling agent (MAPP).

p	$\hat{t}_p$ = quantile	Interval type	LCL	UCL
0.01	29.8632	Standard	27.6952	31.8418
		Percentile	27.8165	31.9345
		Bias-corrected	27.6759	31.7917
0.05	34.7900	Standard	33.0326	36.4559
		Percentile	33.1204	36.5197
		Bias-corrected	33.0440	36.4497
0.10	37.2289	Standard	35.6665	38.7309
		Percentile	35.7217	38.7680
		Bias-corrected	35.6899	38.7143
0.50	44.4709	Standard	43.5488	45.4003
		Percentile	43.5047	45.3792
		Bias-corrected	43.4962	45.3753

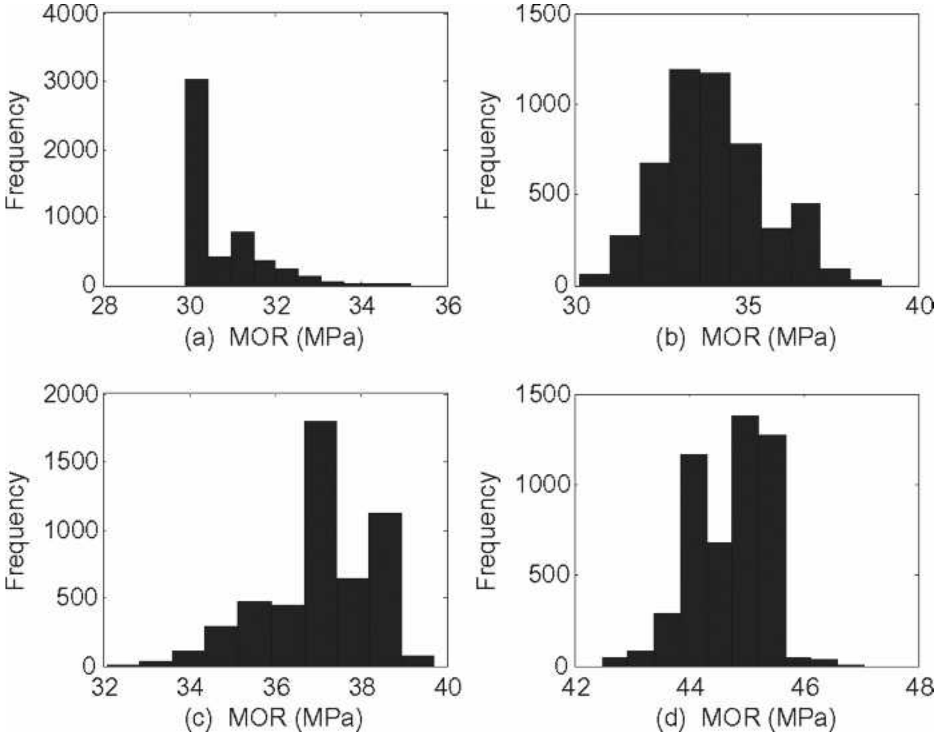

Figure 4. — Sampling distribution of percentiles for WPC MOR with coupling agent (MAPP) under the fully nonparametric bootstrap sampling method. (a) 1st, (b) 5th, (c) 10th, and (d) 50th.

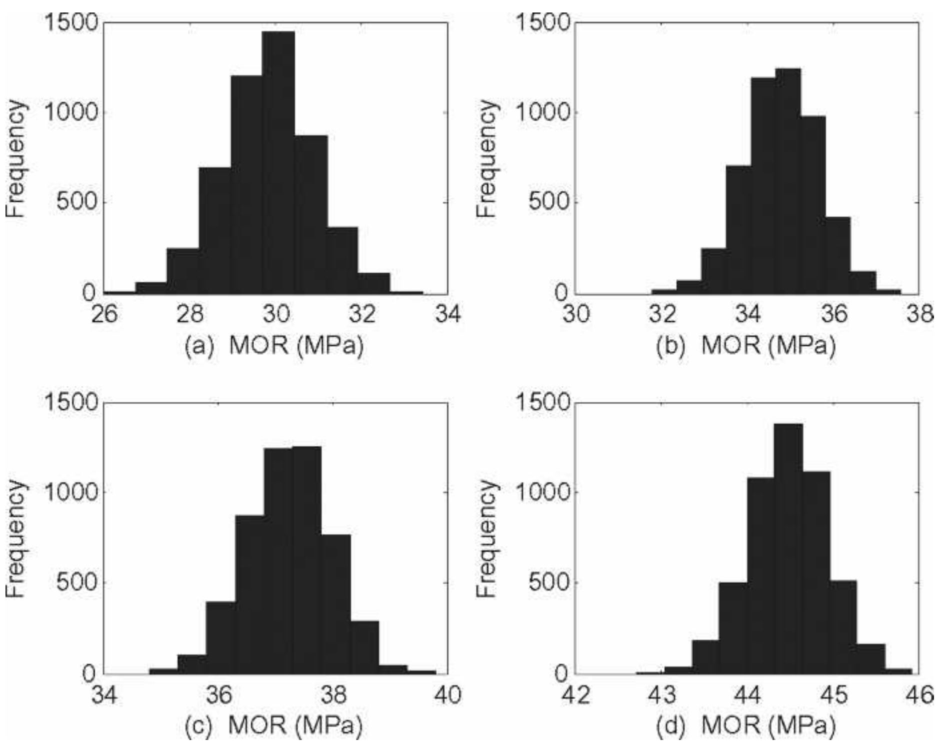

Figure 5. — Sampling distribution of percentiles for WPC MOR with coupling agent (MAPP) under the fully parametric bootstrap sampling method. (a) 1st, (b) 5th, (c) 10th, and (d) 50th.

The best underlying parametric distribution for the MOR of WPC was previously determined as Weibull followed by the smallest extreme value and normal distributions (Perhac 2007, Perhac et al. 2007). Similarly to the MOE data, this best model determination was used to construct the confidence intervals based on both the parametric and the NBSP methods.

Fully parametric 95 percent bootstrap confidence intervals for the MOR are shown in **Table 7**. Widths of the fully parametric intervals are more consistent within each percentile and are more representative of the asymptotic normal confidence interval widths. As the percentiles increase, the distribution widths decrease. As shown in **Table 7**, confidence interval widths are consistent for the standard, percentile, and bias-corrected intervals of each percentile. The interval widths also decrease with increasing percentile. For example, the standard interval for the first percentile ranges from 27.74 to 31.79 MPa. The standard intervals for the fifth, 10th, and 50th percentiles are [33.08, 36.41], [35.74, 38.65], and [43.58, 45.37] MPa, respectively. Average interval differences and the SDs of the three interval types for the 1st, 5th, 10th, and 50th percentiles are 4.024±0.026, 3.311±0.027, 2.920±0.011, and 1.799±0.010, respectively. As was shown with the MOE data, **Figure 5** shows that the sampling distributions for the fully parametric bootstrap sampling method for the MOR data are more continuous and much more normally distributed than for the fully nonparametric bootstrap method for the MOR data.

The NBSP bootstrap method intervals for the MOR data are shown in **Table 8**. These intervals are very similar to those of the fully parametric method with respect to the LCL and UCL limits. In addition, as with the fully parametric method, interval widths for each of the percentiles for the three different types of intervals are very close. As shown in **Table 8**, confidence interval widths are consistent for the standard, percentile, and bias-corrected intervals of each percentile. Similar to the fully parametric bootstrap confidence intervals, the NBSP method interval widths also decrease with increasing percentile. For example, the standard interval for the first percentile ranges from 27.70 to 31.84 MPa. The standard intervals for the fifth, 10th, and 50th percentiles are [33.03, 36.46], [35.67, 38.73], and [43.55, 45.40] MPa, respectively. Average interval differences and the SDs

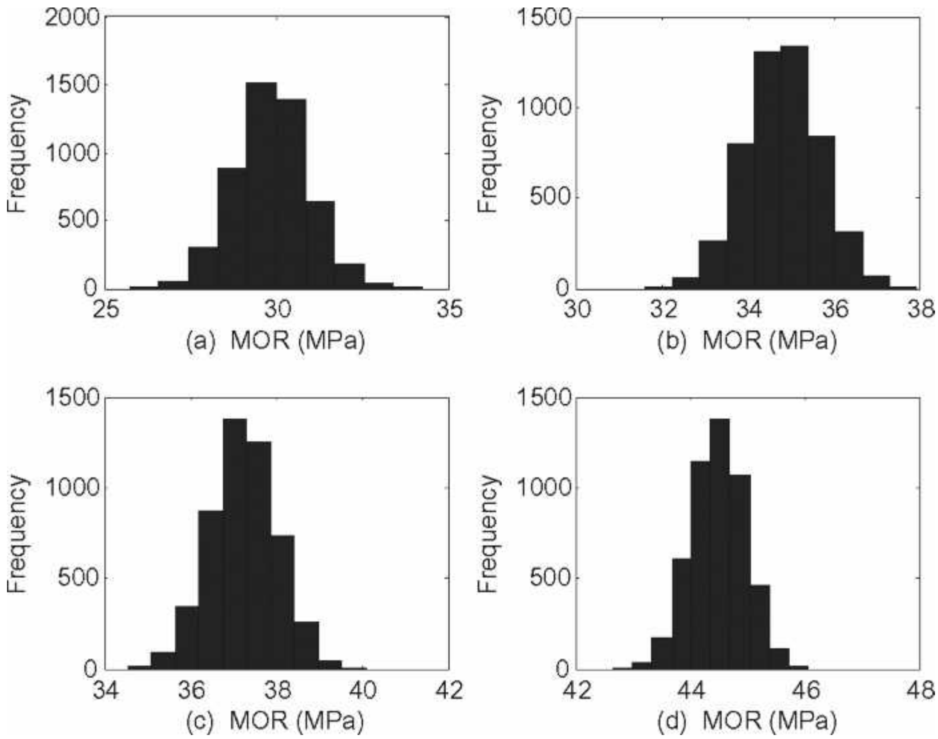

Figure 6. — Sampling distribution of percentiles for WPC MOR with coupling agent (MAPP) under the NBSP method. (a) 1st, (b) 5th, (c) 10th, and (d) 50th.

of the three interval types for the 1st, 5th, 10th, and 50th percentiles are 4.127±0.017, 3.409±0.012, 3.045±0.020, and 1.868±0.015, respectively. As was shown with the MOE data, **Figure 6** shows that the sampling distributions for the NBSP sampling method for the MOR data are very similar to the sampling distributions for the fully parametric method. Both the fully parametric and the NBSP method sampling distributions appear continuous and normally distributed; the main difference between the two methods is the slight skewness shown in the lower percentiles for the NBSP method. **Figure 6** displays the mild right skewness of the first and 10th percentiles, while mild left skewness is seen for the fifth percentile. Because the skewness is so mild and the interval results are so similar to the fully parametric method, the practitioner should certainly consider the NBSP as a viable bootstrapping method when the censoring mechanism cannot be assumed and the sample size is small.

Conclusions

Three different bootstrapping methods were explored. These include the fully nonparametric, fully parametric, and nonparametric bootstrap for parametric (NBSP) models. Three different types of confidence intervals were used to assess four different quantiles for each of the three bootstrapping methods. The confidence interval types included the standard, percentile, and bias-corrected intervals, which were applied to the estimation of the 1st, 5th, 10th, and 50th percentiles of both the MOE and MOR for WPC.

Because the sample sizes used in this analysis are not huge (< 200), using the fully nonparametric bootstrapping method is less desirable than the fully parametric or NBSP methods for smaller percentiles, i.e., lower percentiles have severe right skewness. For some sample sizes, sampling distributions of the lower percentiles tend to be more discrete and far from normally distributed. For the fully parametric and NBSP methods, confidence intervals became narrower as the percentiles increased. This was not the case for the MOE and MOR of WPC for the fully nonparametric method. For this method, no consistent pattern for interval width emerged. Confidence interval widths are also more dissimilar for the fully nonparametric method when using the standard, percentile, and bias-corrected interval types, especially for the MOE data. This dissimilarity and lack of normality complicates the interpretation of these intervals, but helpfully gives warning regarding our confirmation of when such intervals are not robust. Therefore, the fully nonparametric bootstrapping method is not recommended for small sample sizes, particularly for the lower percentile estimations, where the distributions are naturally more discrete. When the underlying distribution is not known, and the sample size is reasonable, utilizing the fully nonparametric method is a valid bootstrapping method (Edwards 2004, Edwards et al. 2008).

If the underlying distribution is not known and the sample size is rather small, the fully nonparametric method may be used to obtain approximate confidence intervals for the median and third quantile. For the very small quantiles, however, this approach is not recommended. One option, in the event of small sample sizes, is to use kernel smoothing to estimate the lower percentiles (Meeker and Escobar 1998, Chernick 1999, and Polansky 2000). When prior information or previous experience is available, another option is to utilize a Bayesian approach to obtain reasonable estimates on the lower percentiles for small data sets (Meeker and Escobar 1998). Both the fully parametric and NBSP bootstrapping methods provide excellent results when the underlying distribution is known. While the fully parametric method provides slightly greater precision and is less susceptible to any apparent skewness, the main advantage of using the NBSP method is that no censoring mechanism knowledge is necessary. The LCL and UCL limits and the interval widths using the NBSP method are remarkably close to that of using the fully parametric method. In addition, the three interval types are similar in consistency with the fully parametric method. In fact, helpfully, for all three bootstrapping methods, the type of interval does not make much difference. This reasonably confirms our confidence interval values when all the intervals are close. The fully parametric method is valuable for validating classical textbook results, but it does not seem to provide a noteworthy advantage over the NBSP method in this data. Recall, the fully parametric method cannot be used when the censoring technique is unknown. For estimating the reliability of WPC, it appears that the bootstrapping method of choice given this data may be the NBSP method. Normality of the bootstrap sampling distributions should always be ascertained by inspecting the histograms, quantile plots, log likelihood estimates, etc., to prevent misuse.

Meeker and Escobar (1998) also recommended that between 2000 and 5000 bootstrap samples be generated in order to compute confidence intervals and that the larger samples are necessary for estimating the lower quantiles, particularly for small samples. Future study may perhaps include investigating the three bootstrapping methods on manufactured WPC data sets with much larger sample sizes or smaller samples using a Bayesian approach.

Bootstrapping is a powerful and effective means of providing estimates of the reliability of WPC that do not have an underlying assumption of normality or other parametric distributions. Such parametric assumptions can lead to greater risk by the practitioner in the estimates of reliability of WPC. Estimation of the lower percentiles allows practitioners to better understand the early failures of their product, i.e., reliability. This understanding facilitates process improvements resulting in better warranties and specification limits for WPC. These process improvements ultimately lead to the production of a superior engineered wood product with enhanced reliability and better customer satisfaction.

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