

*International Journal of*

# **Reliability**

and

# **Applications**

*Volume 7*

**Number 1**

**June 2006**

The Korean Reliability Society [www.koras.or.kr](http://www.koras.or.kr)



International Journal of Reliability and Applications  
Vol. 7, No. 1, pp.27-39, 2006

## **Applying a Forced Censoring Technique with Accelerated Modeling for Improving Estimation of Extremely Small Percentiles of Strengths**

**Weiwei Chen**

*Department of Statistics, 331 Stokely Management Center,  
University of Tennessee, Knoxville, TN 37996-0532 USA*

**Ramón V. León**

*Department of Statistics, 337 Stokely Management Center,  
University of Tennessee, Knoxville, TN 37996-0532 USA*

**Timothy M. Young\***

*Tennessee Forest Products Center, 2506 Jacob Drive,  
University of Tennessee, Knoxville, TN 37996-4570 USA*

**Frank M. Guess**

*Department of Statistics, 338 Stokely Management Center,  
University of Tennessee, Knoxville, TN 37996-0532 USA*

**Abstract.** Many real world cases in material failure analysis do not follow perfectly the normal distribution. Forcing of the normality assumption may lead to inaccurate predictions and poor product quality. We examine the failure process of the internal bond (IB or tensile strength) of medium density fiberboard (MDF). We propose a forced censoring technique that closer fits the lower tails of strength distributions and better estimates extremely smaller percentiles, which may be valuable to continuous quality improvement initiatives. Further analyses are performed to build an accelerated common-shaped Weibull model for different product types using the JMP® Survival and Reliability platform. In this paper, a forced censoring technique is implemented for the first time as a software module, using JMP® Scripting Language (JSL) to expedite data processing, which is crucial for real-time manufacturing settings. Also, we use JSL to automate the task of fitting an accelerated Weibull model and testing model homogeneity in the shape parameter. Finally, a package script is written to readily provide field engineers customized reporting for model visualization, parameter estimation, and percentile forecasting. Our approach may be more accurate for product conformance evaluation, plus help

---

\* Corresponding Author.  
*E-mail address:* tmyoung1@utk.edu

reduce the cost of destructive testing and data management due to reduced frequency of testing. It may also be valuable for preventing field failure and improved product safety even when destructive testing is not reduced by yielding higher precision intervals at the same confidence level.

**Key Words** : *first percentile, lower percentiles, forced censoring for fitting better, strengths of materials, non-normal data, accelerated model, JMP®, scripting language reliability.*

## 1. INTRODUCTION

The normal distribution is realistically assumed for many applications during the quality improvement process (Meeker and Escobar 1998). However, there are many practical cases where a better fit of the data is from non-normal distributions. Where normality is not appropriate, forcing the normal distribution model can lead to inaccurate prediction of key process parameters and result in poor product quality. Stanard and Osborn (2002) have discussed general strategies for handling non-normality in a “Six Sigma Quality” context. Guess, León, Chen, and Young (2004) have presented a case study, in which the internal bond (IB or tensile strength) of medium density fiberboard (MDF) does not follow perfectly a normal process in the lower tails. The estimation of crucial lower percentiles can be poor when incorrectly assuming the normal distribution and such analytical errors can be very costly for manufacturers, compare Guess, Edwards, Pickrell and Young (2003).

Guess, León, Chen, and Young (2004) propose the median censoring technique in modeling extremely small failure data and estimating the lower percentiles of strengths. All the observations no larger than the median are retained intact as exact failures, while observations beyond the median are censored at a forced value slightly larger than the median but less than the next true observed failure above the median. After applying the censoring technique a better fit is found in the lower tails, where the smaller percentiles are impacted the most. The analysis indicates that the Weibull distribution fits the lower strength MDF better, while the overall strength is better fitted by the normal distribution.

The central idea of the forced censoring technique is to preserve as much useful information as possible in the raw data and to extract desired local information from leveraged data. This is a very useful technique when data is complex in nature and the data collection is expensive. This forced censoring technique is different from other known strategies such as truncation, Box-Cox transformation, or segmentation, when working with non-normal data. The complexity of data structure, like multiple failure modes, is well respected and captured as a whole even when estimating a local parameter. In Section 2 we elaborate on a mechanism of forced censoring technique and give more discussions on the potential applications of this technique. Section 3 describes the statistical discovery of a Weibull common shape parameter and modeling using SAS product JMP®. In Section 4, we construct a log likelihood ratio test yielding quantitative evidence of a Weibull common shape parameter, following similar procedures when

building an accelerated model. Furthermore, JMP scripting is provided to help field engineers automate the analysis and reporting routines. Section 5 concludes the paper.

## 2. FORCED CENSORING VIA JMP® SCRIPTING

We can further extend the median censoring technique to any percentile of a data set. Employing the power of JMP® Scripting Language (JSL), we scripted a module in JMP® that automatically force-censors the data from any percentile point of interest. More specifically, the implementation of this JMP® script is to replace the observations larger than a specified percentile value with this new percentile value, and label the replaced observation as “censored”. Note that in JMP® by default, censor label values of zero indicate the event (e.g., 0: failure), and a non-zero (e.g., 1: able to customize) code is a censored value. Recall JMP® (<http://www.jmp.com>, a SAS® division) is statistical discovery software for data analysis, DOE, Six Sigma Quality, and other applications, while also featuring interactive graphic exploration with scripting flexibility (SAS Institute, Inc. 2004). Other software could be programmed to do this, also.

The helpful script of JSL-implemented forced censoring can be found in our Appendix. Below are illustrations of the screen shots of interactive JMP® dialog before censoring and an example data table useful for further modeling.

Figure 1 consists of two parts: (a) a JMP dialog box and (b) a data table. Part (a) shows a dialog box titled 'JMP' with two input fields: 'Enter censor quantile' with the value '0.5' and 'Enter product type' with the value 'Type 1'. There are 'OK' and 'Cancel' buttons. Part (b) shows a data table with columns: IB, Censor, IBC, NewCensor, and Type. The data is as follows:

	IB	Censor	IBC	NewCensor	Type
1	129.000000	0	120.200005	1	Type 1
2	120.399994	0	120.200005	1	Type 1
3	124.300003	0	120.200005	1	Type 1
4	132.699997	0	120.200005	1	Type 1
5	122.699997	0	120.200005	1	Type 1
6	127.199997	0	120.200005	1	Type 1
7	134.199997	0	120.200005	1	Type 1
8	108.199997	0	108.199997	0	Type 1
9	129.500000	0	120.200005	1	Type 1
10	127.599998	0	120.200005	1	Type 1
11	117.500000	0	117.5	0	Type 1
12	119.300003	0	119.300003	0	Type 1
13	137.200012	0	120.200005	1	Type 1
14	141.199997	0	120.200005	1	Type 1
15	138.199997	0	120.200005	1	Type 1
16	110.300003	0	110.300003	0	Type 1
17	114.300003	0	114.300003	0	Type 1
18	125.500000	0	120.200005	1	Type 1
19	127.800003	0	120.200005	1	Type 1
20	122.800003	0	120.200005	1	Type 1
21	123.099998	0	120.200005	1	Type 1
22	113.000000	0	113.000000	0	Type 1

a) JMP® dialog asking for customized censor quantile

b) Data prepared for further analysis

**Figure 1.** Screen illustrations of forced censoring implemented in JMP®

The right-censoring mechanism is sufficient in our case study of extremely small percentiles. Other product or weather applications may require modeling the upper part or an intermittent portion of data. For example, a process engineer may want to estimate the number of particles on a silicon wafer, which leads to defective computer chips. Both the

small and large percentiles of the distribution of particle numbers per wafer would be key indicators of the quality of the production run. The normal probability plot might show a severe departure from the straight line on both the lower and upper parts of the distribution. Further analysis may reveal inherently non-normal data with no known simple distribution function yielding satisfactory estimates to the key percentiles on either end of the distribution. Different portions of the distribution would need to be examined by themselves in such a complex case. Observations may be treated as either right-censored, left-censored, interval-censored, or remain entirely uncensored depending on analytical needs. The example script (Appendix) can be used to implement a modified all-purpose forced censoring mechanism in JSL. All three types of censoring mechanism, right, left, or interval can be customized in the format of interval censoring. More detail is given in JMP® Manual: Statistics and Graphics Guide, section “Interval Censoring” in Topic titled “Survival and Reliability Analysis”.

### 3. DISCOVERING COMMON SHAPE PARAMETER

We have applied the forced censoring technique to two important MDF product types. In Guess, León, Chen and Young (2004), a MDF product type of high consumer usage, (“Type 1”) was investigated. We currently investigate a second important product (“Type 5”) and make comparisons between both products. The main difference between the two product types is density

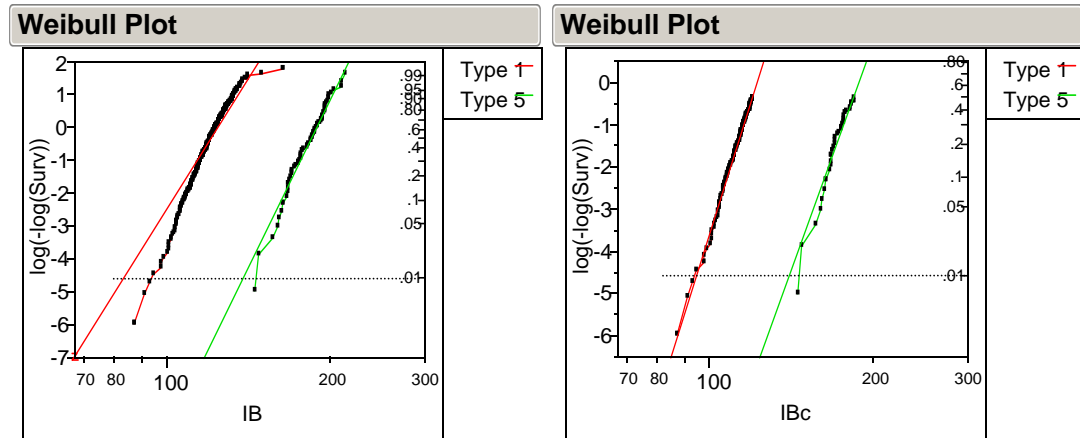
**Table 1.** Key specifications of Types 1 and 5.

Type	Density	Thickness	Width	Sample #	Note
1	A	same	same	396	Standard density
5	B	same	same	74	High density

Type 1 is more popular in terms of demand and production volume while the Type 5 product provides more value with higher density. Both types are of great commercial interests to the manufacturer and consumer. The production cost of Type 5 is higher than Type 1.

A sound statistical analysis always starts with graphical explorations of the data. We now take advantage of the powerful graphics provided by the JMP® Survival/Reliability platform and present comparisons of two product types side by side in the next several Figures 2 and beyond.

A few important observations can be made from the graphical analysis presented in Figure 2. First, the forced censoring technique provides a closer fit to the focus portion of data for Type 1. Second, there is departure on the lower tail of Type 5 which the median censoring does not improve upon as much.



a.) Weibull fit to uncensored data of both product types

b.) Weibull fit to the median censored data of both product types

**Figure 2.** Comparisons of Type 1 and Type 5 on the Weibull probability plots in JMP®

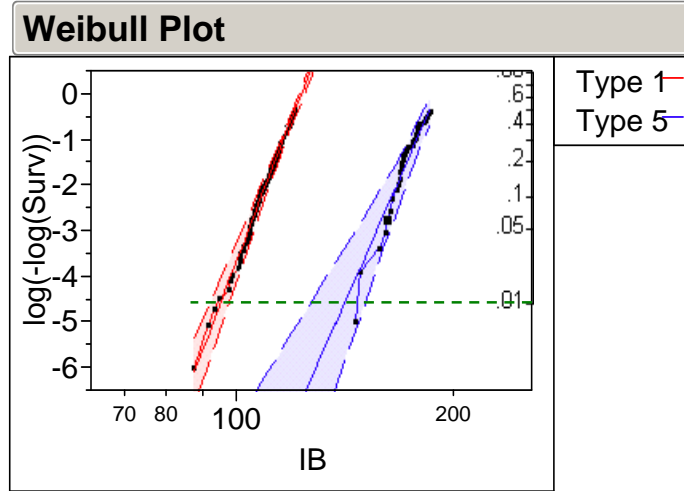
Note, the manufacturing process for these MDF products is not a batch process and is continuous flow i.e., there is a gradual transition from Type 1 to another Type. The variations observed in the upper percentiles of Type 1 and lower percentiles of Type 5 may likely be the result of this gradual production transition phase. We have successfully applied the median censoring technique to Type 1 to reduce the upper-tail influence in the case of modeling lower percentiles. In Type 5 an undesired outcome is that the lower-tail variance might affect somewhat our estimation on the small percentiles.

A practical strategy is to have a relatively conservative interval estimate of the percentile. Figure 3 illustrates the 95% simultaneous confidence interval for both types of products, generated in JMP® 6.0 beta test version. The JMP® 6.0 public version is planned to be out sometime by November or December, 2005.

In both Figures 2 and 3, the median censoring technique helpfully improves the fit of the lower tail of Type 1 product to the Weibull model. For Type 5 product, even though Figure 2 does not show as much difference between the fits of uncensored data and censored data of Type 5 product, it can be seen in Figure 3 that the Weibull fit to median-censored Type 5 product data renders a relatively wider confidence interval that realistically accommodates the rather large variations on the lower tail that is inherent there with the smaller sample. In fact, had the Type 5 data not been censored, one crucial lower percentile data point would be beyond the Weibull 95% confidence bands of the uncensored Type 5 product data (not shown in Figure 3). Based on the above reasoning, we prefer the more conservative results from the censored Type 5 data.

Recall that the censoring technique does not truncate the data; rather the data portion of interest is given more weight for modeling. In the case of Type 5, the result is that more leeway is given to the lower percentile estimate given the relatively large local variations. (Aside: The plot option of fitted confidence interval is a newly added feature of JMP® Survival/Reliability Platform in the beta version 6.0 we are reviewing). Another

important discovery from the graphical analysis (Figures 2 and 3) is the surprising similarity of failure modes between Types 1 and 5. The rest of this section explains in further details the rationale underlying our graphical exploration.



**Figure 3.** 95% Simultaneous Confidence Intervals of median-censored Type 1 and 5 products on the Weibull probability plots

Recall the Weibull model and linearized Weibull probability plot. The Weibull CDF can be often written as

$$P(T \leq t; \alpha; \beta) = 1 - \exp\left[-\left(\frac{t}{\alpha}\right)^\beta\right], t > 0.$$

$\beta > 0$  is the shape parameter and  $\alpha > 0$  is the scale parameter as well as 0.632 quantile (Weibull 1939, 1951). Meeker and Escobar (1998) have pointed out that the practical value of the Weibull distribution is to describe failure distributions with many different commonly occurring shapes. To better display or compare parametric models such as Weibull, we linearize a model CDF on the probability plot. In the Weibull case, one can derive the  $p$  quantile from the above CDF function:  $t_p = \alpha[-\log(1 - p)]^{1/\beta}$ . This leads to

$$\log(t_p) = \log \alpha + \log[-\log(1 - p)] \frac{1}{\beta}$$

If we use special scales to  $t_p$  and  $p$  on the probability, which is to take  $\log t_p$  and  $\log[-\log(1 - p)]$  on the x and y axis, there is a linear relationship between  $\log[-\log(1 - p)]$  and  $\log t_p$  provided a perfect Weibull distribution where the shape parameter  $\beta$  is the slope of the straight line. This justification underlies all the Weibull probability plots shown so far. Hence, if two models appear to have similar slopes on the

Weibull probability plots, we may hypothesize that the two models have the same shape parameter, which is also an indicator of failure mode.

In our case study, the Weibull probability plots for Types 1 and 5 failure modes have similar slopes (Figures 2 and 3). Before median censoring, the Weibull model was fit to the data of each product type. The model estimates are shown in Table 2. Note that the 95% confidence intervals for the shape parameter  $\beta$  of each product type do not even overlap. However, a refit of the Weibull model to the median censored Types 1 and 5 failure data produced similar range of confidence interval estimates of the shape parameter  $\beta$  (Table 3).

**Table 2.** Weibull parameter estimates based on uncensored Types 1 and 5 data.

Product type	Parameter	Estimate	Lower 95%	Upper 95%	N Tests
Type 1	$\alpha$	124.76	123.61	125.90	396
Type 1	$\beta$	11.38	10.66	12.10	396
Type 5	$\alpha$	190.03	186.83	193.18	74
Type 5	$\beta$	14.60	12.18	17.22	74

**Table 3.** Weibull parameter estimates based on median censored Types 1 and 5 data.

Product type	Parameter	Estimate	Lower 95%	Upper 95%	N Tests
Type 1	$\alpha$	122.71	121.70	123.87	198
Type 1	$\beta$	17.79	15.59	20.18	198
Type 5	$\alpha$	189.51	185.73	194.88	37
Type 5	$\beta$	15.55	11.38	20.57	37

The closeness of individual model parameters alone is not sufficient to conclude that the two product types have the same shape parameters, or the same type of failure modes. Therefore, we conducted an additional rigorous statistical investigation to determine whether the two products had a common shape parameter (similar to the strategy for analyzing accelerated life test data). We view density for each product type, or simply a dummy variable indicating the product type, as the accelerated variable in the next Section. Compare Meeker and Escobar (1998).

#### 4. ACCELERATED WEIBULL MODELS FOR TYPES 1 AND 5

We explore a constant-shape parameter assumption that can lead to an overall constrained Weibull model  $\{\alpha_1, \alpha_2, \text{ and common shape } \beta\}$  for the replacement of two individual unconstrained Weibull models  $\{W1: \alpha_1, \beta_1\}$  and  $\{W2: \alpha_2, \beta_2\}$ . The total likelihood for the unconstrained models is always larger than the likelihood of the constrained model. A likelihood ratio test is conducted to determine if the total likelihood

for the unconstrained models is large enough to indicate lack of fit for the constrained model. The null and alternate hypotheses for the likelihood ratio test are:

$H_0$  : The shape parameters are the same.

$H_1$  : The shape parameters are different; the unconstrained models are better.

The test statistic  $Q = -2(L_{constrained} - L_{unconstrained}) = -2[L_{constrained} - (L_{w_1} + L_{w_2})]$ ,  $L$  denoting the log likelihood of each model, follows a  $\chi^2_1$  distribution, in which the one degree of freedom comes from the difference between the number of parameters in constrained and unconstrained models.

An individual survival model is built using JMP® “Fit Parametric Survival” platform from its “Survival and Reliability” submenu. In the accelerated Weibull model, we include the accelerating variable (dummy variable: product type) as the regressor or “model effect” with the model specified. Table 4 illustrates the log likelihood values from three models:  $L_{w_1}$ ,  $L_{w_2}$ ,  $L_{constrained}$  and the chi-square test results.

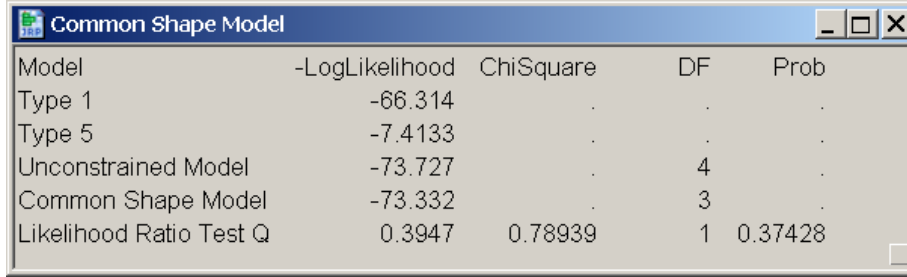
**Table 4.** Demonstration of likelihood ratio test based on JMP® “Fit Parametric Survival” output with the forced censored data

Model	Log likelihood	Chi Square	d.f.	Prob>ChiSq
W1 (Type 1)	-66.3138			
W2 (Type 5)	-7.4132			
Unconstrained (W1+ W2)	-73.7271		4	
Constrained (common shape $\beta$ )	-73.3324		3	
Test Statistic Q		0.7894	1	0.3744

The estimated value of  $Q = -2 \times [-73.3324 - (-66.3138 - 7.4132)] = 0.7894$  is less than the critical  $\chi^2_{0.95,1} = 3.84$  (p-value = 0.3744), indicating not enough evidence against inadequacy of the constrained model. Based on this test, there is insufficient evidence to reject the null hypothesis that shape parameters for Types 1 and 5 were the same. Another way to check the model adequacy is to use Akaike’s Information Criterion (AIC):  $-2L + 2k$ ,  $k$  being the number of parameters in the model, Akaike (1973). The conclusion was the same using the AIC test, i.e., the common shape model is adequate for modeling.

Constructing data tables, fitting separate models, and extracting log likelihood results from different reports for statistical testing can be very tedious and may be subject to human error even when an easy-to-use interactive interface such as JMP® is used without

scripting. We develop a JMP® script to automate the data preparation and model computing process to complement graphical exploration and model building (see Figure 4 JMP® output of a customized report of likelihood ratio test for common shape model using our customized JSL). See Young and Guess (2002) for more on process automation and how data is stored and used in a real time data base with regression modeling to predict strength. Also, see English (1999) on designing a high information quality model for less information scrap and rework.



Model	-LogLikelihood	ChiSquare	DF	Prob
Type 1	-66.314	.	.	.
Type 5	-7.4133	.	.	.
Unconstrained Model	-73.727	.	4	.
Common Shape Model	-73.332	.	3	.
Likelihood Ratio Test Q	0.3947	0.78939	1	0.37428

**Figure 4.** Customized JMP® Report of Likelihood Ratio Test for Common Shape Weibull Model

We are more interested in this investigation of the practical implications suggested by the common shape model associated with the Weibull distribution. For this data, Type 5 was a product of high value to the producer and consumer but is not sampled at the same level of intensity as Type 1, another important product. To understand the confidence in the estimates for Type 5 key parameters for the common shape Weibull model we investigated several methods to ensure product reliability. Given that dissimilar sample sizes of Types 1 and 5, we use the abundant information from Type 1 to assist in the model building and prediction of Type 5. A comparison of the various percentile estimates for each product is presented in Table 5

**Table 5.** 95% confidence intervals of first percentiles computed under various model assumptions with and without the median censoring technique.

a) Type 1 product

Model Assumption	With median censoring		W/O median censoring		Interval Method
	95%_Lower	95%_Upper	95%_Lower	95%_Upper	
Weibull	91.834	97.392	88.085	97.164	Bootstrap-t *
Weibull	91.206	98.424	81.276	85.312	JMP® Individual Model
Weibull	90.886	97.656	82.359	86.061	JMP® Common Shape Model

## b) Type 5 product

Model Assumption	With median censoring		W/O median censoring		Interval Method
	95%_Lower	95%_Upper	95%_Lower	95%_Upper	
Weibull	139.6	154.46	130.38	148.54	Bootstrap-t *
Weibull	127.31	155.60	131.71	146.00	JMP® Individual Model
Weibull	139.36	150.79	123.60	131.07	JMP® Common Shape Model

\* Rows with \* are bootstrapped estimates, as comparisons to JMP® results. See Meeker and Escobar (1998)

Table 5 shows consistent confidence interval estimates from the bootstrapped and JMP® common shape models, for both product types after being median censored. One exception is the estimate for product Type 5 from the JMP® individual model. Recall the relatively large variations on the lower tail of Type 5 in Figures 2 and 3 due to production transition phase. The JMP® common shape model performs as well as the bootstrap method, even though the methodologies are different. However, because of the relatively large variation right at the percentile point of interest, more evidence and cross-validation results are needed to enhance our confidence in recommending one of these estimates.

## 5. CONCLUSION

In conclusion, we have presented more evidence that the forced censoring technique enhances the analysis of data when we are interested in lower quantiles of the distribution. (Similar strategies can be used for upper quantiles or other parts.) We further investigated the assumption of using a common shape parameter for the Weibull model for the two types of fiberboard to help increase the accuracies of extremely small percentile estimates. This may also be important methods for understanding product reliability and be helpful for improved product quality and lower manufacturing costs. The easy-to-use JMP® platform facilitated the implementation of a sound statistical strategy in the context of process improvement in reliability engineering that can be readily adopted by a large number of industrial users

## ACKNOWLEDGEMENTS

Timothy M. Young, Weiwei Chen, and Frank M. Guess gratefully acknowledge funding in 2004-2005 from the United States Department of Agriculture, Special Wood Utilization Grants Program as administered by The University of Tennessee Institute of Agriculture Experiment Station, contract R11-2219-109.

Weiwei Chen is thankful for an honorarium awarded by SAS and the opportunity of presenting work at the 2nd Annual JMP® Conference, June 7-8, 2005, SAS World

Headquarters, Cary, North Carolina, U.S.A.

## REFERENCES

- Akaike, H. (1973). Information Theory and an Extension of the Maximum Likelihood Principle. In B.N. Petrov and F. Csaki (eds.), *Second International Symposium on Information Theory*, Akademiai Kiado, Budapest, 267-281.
- English, L. P. (1999). *Improving Data Warehouse and Business Information Quality: Methods for Reducing Costs and Increasing Profits*. Wiley, New York, NY.
- Guess, F. M., Edwards, D. J., Pickrell, T. M. and Young, T. M. (2003). Exploring Graphically and Statistically the Reliability of Medium Density Fiberboard. *International Journal of Reliability and Application*, **4** (4), 97-109.
- Guess, F. M., León, R.V., Chen, W., and Young, T. M. (2004). Forcing a Closer Fit in the Lower Tails of a Distribution for Better Estimating Extremely Small Percentiles of Strengths. *International Journal of Reliability and Application*, **5** (4), 129-145.
- Meeker, W. Q. and Escobar, L. A. (1998). *Statistical Methods for Reliability Data*. Wiley, New York, NY.
- SAS Institute Inc. (2004). JMP® 5.1 Manual, SAS Publications.
- Stanard, C. L. and Osborn, B. E. (2002). *Six Sigma Quality Beyond the Normal*. GE Global Research Technical Report, 2002GRC119.
- Weibull, W. (1939). A statistical theory of the strength of materials, *Ing. Vetenskaps Akad. Handl.*, 151 (1), 1-45
- Weibull, W. (1951). A statistical distribution function of wide applicability, *Journal of Applied Mechanics*, **18** (1), 293-297.
- Young, T. and Guess, F. (2002). Mining Information in Automated Relational Databases for Improving Reliability in Forest Products Manufacturing. *International Journal of Reliability and Application*, **3**(4), 155-164.

**APPENDIX**

JMP® script of the forced censoring technique:

```
-----
dt = Open();
/* Dialog to choose the censoring quantile */
sdlg = Dialog(
    "Enter censor quantile",
    censorQt = EditNumber(0.50),
    "Enter product type",
    pType=EditText("Type 1"),
    Button("OK"),
    Button("Cancel")
);
If (sdlg["Button"]!=-1,
    Throw("Cancelled")
);
show(sdlg["censorQt"]);
show(sdlg["pType"]);

censorValue = Col Quantile(:IB, sdlg["censorQt"]);

/* Create new columns for censored data, censor label, and product type info. */
dt << New Column ("IBc",
    Numeric,
    Continuous
);
dt << New Column ("NewCensor",
    Numeric,
    Nominal
);
dt << New Column ("Type",
    Char,
    Nominal
);

/* Forced censoring from the specified quantile for each row */
For Each Row (
    If(
        :IB <= censorValue, :NewCensor=0; :IBc=:IB,
        :IB > censorValue, :NewCensor=1; :IBc=censorValue
    );
    :Type=sdlg["pType"]
);
```

```
/* Create new data table; not overwrite the initial data file */  
dtnew = dt << Subset(  
    Output Table(sdlg["pType"]||" censored"),  
    Columns(:IB, :Censor, :IBc, :NewCensor, :Type)  
);  
  
close(dt, no save);  
close(dtnew);  
-----
```